N0AX

HANDS-ON RADIO

Experiment #81 — Synchronous Transformers

Why are these transmission line sections *synchronous*? Synchronous has multiple definitions, but the one that applies here is: *going on at the same rate and exactly together; recurring together* — from *syn* (together with) + *chron* (time) + *ous* (possessing or having the quality of). In this case, time refers to phase. Specifically, the phase of waves in the transmission line used to make up this month's project!

Quarter Wave Transformer (Q section)

The best-known example of a synchronous transformer in Amateur Radio is the *quarter wave transformer*, also known as a *Q section*, shown in Figure 1. The Q section creates an impedance match between two impedances, Z_1 and Z_2, by inserting between them a quarter-wavelength of transmission line with a characteristic impedance that is the geometric mean of the impedances to be matched, $Z_Q = \sqrt{(Z_1 \times Z_2)}$.

That deceptively simple equation represents what happens as the result of an infinite series of reflections occurring at the junctions of the three sections of transmission line. Figure 2 illustrates the first few steps. Let's follow along. Beginning in Step 1, as the electromagnetic wave in Line 1 encounters transmission lines with impedance different than that in which it's traveling.

From transmission line theory, we know that some of the incident wave's energy will be reflected at the impedance discontinuity (Step 2), generating a reflected wave. Viewing the waves in terms of voltage, the incident wave is E_{I1} and the reflected wave is E_{R1}. The ratio between the incident and reflected voltages is the *reflection coefficient*, $\rho_1 = E_{R1} / E_{I1}$. That means $E_{R1} = \rho_1 \times E_{I1}$.

If the wave encounters an infinite or zero impedance (an open or short circuit, respectively), $\rho = 1$. If the impedance the wave encounters is the same as the impedance of the line it's traveling through, such as a matched load, there is no reflection and $\rho = 0$. For any other value of impedance encountered, ρ is between 0 and 1. (The full representation of ρ includes phase, meaning that ρ is really a complex number of the form $|\rho| \angle\theta$, but for the purposes of this discussion, we will consider the magnitude and phase of ρ separately.)

The reflected wave, E_{R1}, travels back along Line 1 in the opposite direction from the incident wave. The remaining energy that wasn't reflected continues on in Line 2 with a new voltage, E_{I2}. Whatever generated the original wave, E_{I1}, will eventually see the reflected wave, E_{R1}, return.

That might be the end of the story, but there is another change in impedance a little further along where Line 2 meets Line 3, and another set of reflections is generated (Step 3), with E_{I3} continuing on in Line 3. If the reflection coefficient at the second discontinuity is ρ_2, then another reflected wave, E_{R2}, is generated with a voltage of $\rho_2 E_{I2}$, traveling in the same direction as E_{R1}. It encounters the initial discontinuity

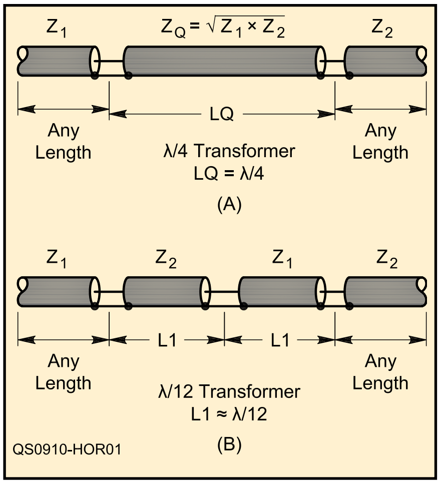

Figure 1 — The ¼ wave and $^1/_{12}$ wave synchronous transformers. A series of carefully phased reflections create an impedance match.

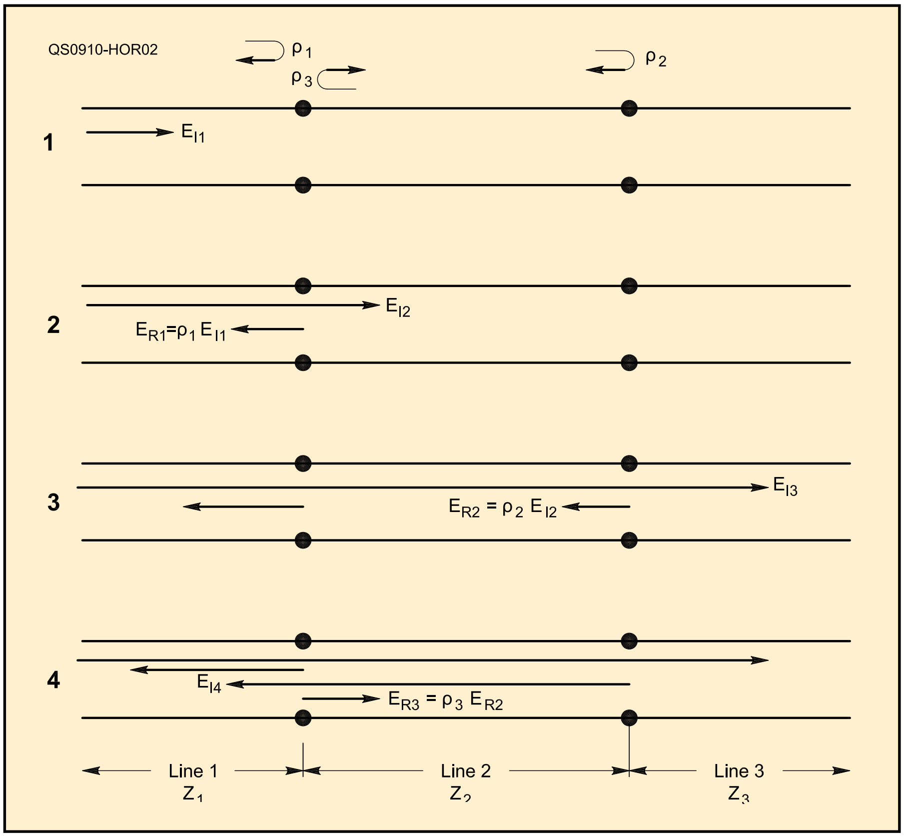

Figure 2 — The initial sequence of reflections generated by a ¼ wavelength section of transmission line inserted between lines of different characteristic impedances.

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(Step 4) while traveling in the opposite direction as the original incident wave, E_I, and generates another pair of waves, E_{I4} and E_{R3}, according to reflection coefficient ρ_3.

We now have three waves adding together in Line 1 — E_{I1}, E_{R1} and E_{I4}. E_{R1} is smaller than E_{I1} and E_{I4} is smaller than E_{R1}. The resulting voltage sum, however, depends on the relative phase of all three waves. The phase difference between E_{I1} and E_{R1} depends on the relationship between Z_1 and Z_2: if $Z_2 > Z_1$, then E_{R1} is in phase with E_{I1}, and out of phase if $Z_2 < Z_1$. (We are assuming all three impedances are purely resistive, so no additional phase shift due to reactance occurs.) E_{I4} differs in phase from E_{R1} by twice the electrical length of Line 2 because the wave has to travel through Line 2 once in each direction before returning to the junction of Lines 1 and 2.

You can see that more reflections will occur as E_{R3} travels back to the junction of Line 2 and Line 3 and generates another pair of waves. This happens forever, until the incident wave E_{I1} ceases. (We are also ignoring any reflections created at the unseen ends of Line 1 and Line 3.) As each set of increasingly smaller reflected and incident waves are combined, the result eventually converges on a steady-state value for the voltages of the waves traveling in each direction in all three segments of transmission line.

In Line 1, this combination of voltages means that whatever is generating the incident wave, E_{I1}, will be presented with an impedance different from Z_1 and the SWR will be greater than 1:1. Or will it?

Given fixed values for Z_1 and Z_3, we can still control how the waves add up in Line 1 by adjusting both the impedance and electrical length of Line 2. Skipping to the punch line, if Line 2 happens to be 90° long and its impedance is the geometric mean of Z_1 and Z_2 as noted earlier, all the reflections in Line 1 cancel, the impedance at the input to Line 1 is Z_1, and the SWR will be 1:1. Behold — the quarter-wave impedance transformer!

The impedance match results from the cancellation because of the precisely timed (thus synchronous) reflections that all add together to cancel all waves traveling in the direction from which E_{I1} comes. Not only that, but the same set of wave mechanics create a match in the other direction, too, so the SWR looking toward the Q section from Line 3 is also 1:1! (Note that I have glossed over a significant amount of mathematics in developing this explanation.[1] You're welcome.)

An exact match occurs only at the frequency for which Line 2 is ¼ wavelength long (or some odd integer multiple of ¼ wavelength) and if losses in Line 2 are low enough that they have an insignificant effect. Nevertheless, the Q section provides an excellent match over several percent of bandwidth — good enough to use for a whole band at and above 7 MHz. Let's make one!

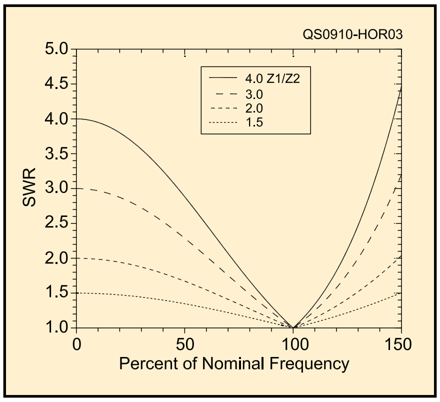

Figure 3 — The bandwidth of the ¹⁄₁₂ wavelength transformer. The resulting SWR for frequencies from dc to 1.5 times the design frequency and resistive impedance mismatches up to 4:1 (from Note 3).

Making a Q Section

The hardest part about making a Q section is often just coming up with the odd impedance transmission line! However, there's no need to obsess over getting an exact match — any impedance within 10% of the exact value will give good results. We'll use 75 Ω coax (RG-59) for our Q section with a design frequency of 14.15 MHz, the middle of the 20 meter band.

The length of a quarter-wave piece of transmission line in feet is VF × 246 / f (in MHz), where VF is the *velocity factor* for the line. Cable with a solid polyethylene dielectric has a VF of 0.66. If your cable has a foam dielectric, the VF is 0.78 to 0.83 (check the manufacturer's Web site for the data sheet). Calculate the length of cable you need (for solid dielectric, that would be 11.5 feet) and cut it about 5% long, or 12.05 feet.

Tune the Q section by shorting one end of the cable (twist the center conductor and shield braid together) and attaching the other end to your antenna analyzer. Ignore the SWR value and tune down in frequency from 28 MHz. Find the lowest frequency at which resistance is a minimum. This is the frequency at which the line is ½ wavelength long, so divide by two to get the ¼ wavelength frequency. Trim an inch or less at a time and repeat until the section is ¼ wavelength long at close to 14.15 MHz.

A Q section using 75 Ω line will match 75^2 / 50 = 112.5 Ω to 50 Ω, so terminate one end of the Q section with two 220 Ω resistors in parallel using reasonably short leads. Verify that the SWR at the other end of the Q section is close to 1:1. Vary the frequency and find the Q section's *SWR bandwidth* — the range of frequencies over which SWR is 2:1 or less. Substitute different values of terminating resistance to find out how much the termination can vary in either direction before SWR at the other end exceeds 2:1. Try the Q section at a frequency where it is ¾ wavelength long and again at ⁵⁄₄ wavelength.

Twelfth Wave Transformers

The Q section is really a special case of series section matching.[2] There's no restriction (other than complexity) that there be just one matching section. In fact, the two section variation shown in Figure 1B is quite handy for matching two different impedances of transmission line, such as 50 Ω coax and 75 Ω hardline.[3] Best of all, it doesn't require any special transmission line impedances, only sections of line with the same impedances that are to be matched!

This configuration is referred to as a *twelfth wave transformer* because when the ratio of the impedances to be matched is 1.5:1, as is the case with 50 and 75 Ω cables, the electrical length of the two matching sections between the lines to be matched is 0.0815 λ, quite close to λ/12 (0.0833 λ). As the lowest line in Figure 3 shows, the SWR bandwidth of the transformer is quite broad, but decreases as the ratio of impedances to be matched increases. You can use this trick to make good use of surplus low loss 75 Ω CATV hardline between 50 Ω antennas and radios!

Parts List

- Coaxial cable, 20 feet of RG-59.
- Resistors, 2 each, 220 Ω, ¼ W carbon composition (non-inductive).

Recommended Reading

Quarter-wave transformers are the basis for non-reflective optical coatings. As you read through **en.wikipedia.org/wiki/Optical_coating** you may recognize several similarities between optics and transmission lines. ARRL members should also download the referenced *QST* articles from the *QST* Archive.

Next Month

How high should it go? Isn't higher better? Find out next month as we do a virtual experiment using antenna modeling software to observe the effects of height above ground on antennas.[4]

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[1] R. Lay, W9DMK, "A Transient Analysis of an Impedance Transforming Device (The Quarter-Wave Transformer)," **www.qsl.net/w9dmk/qrtrwav4.pdf**.

[2] F. Regier, OD5CG, "Series-Section Trans mission-Line Impedance Matching," *QST*, Jul 1978, pp 14-16.

[3] D. Emerson, AA7FV, "Try a Twelfth-Wave Transformer," *QST*, Jun 1997, pp 43-44.

[4] Previous Hands-On Radio columns and a complete parts list for all experiments are available to ARRL members at **www.arrl.org/tis/info/HTML/Hands-OnRadio.**